

# Physically meaningful and not so meaningful symmetries in Chern-Simons theory

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We explicitly show that the Landau gauge supersymmetry of Chern-Simons theory does not have any physical significance. In fact, the difference between an effective action that is both Becchi-Rouet-Stora (BRS) invariant and Landau supersymmetric and an effective action that is only BRS invariant is a finite field redefinition. Having established this, we use a BRS-invariant regulator that defines CS theory as the large mass limit of topologically massive Yang-Mills theory to discuss the shift  $k \rightarrow k + c_V$  of the bare Chern-Simons parameter  $k$  in connection with the Landau supersymmetry. Finally, to convince ourselves that the shift above is not an accident of our regularization method, we comment on the fact that all BRS-invariant regulators used as yet yield the same value for the shift.

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## I. INTRODUCTION

Canonical quantization of three-dimensional Chern-Simons (CS) theory has provided two very interesting results [1]. One is the relation between the vacuum expectation values of the Wilson loops of the theory and the intrinsically three-dimensional characterizations of knot and link invariants. The other one is a framework to understand properties of two-dimensional conformal theory. In both issues, two features of CS theory play a major part: its finiteness and the shift of the bare CS parameter  $k$ ,

$$k \rightarrow k + \text{sgn}(k)c_V, \quad (1.1)$$

$c_V$  being the quadratic Casimir operator in the adjoint representation of the gauge group. For a variety of reasons, one would like to understand these two properties from a perturbative point of view. Among such reasons, we mention first the fact that perturbative quantization has led to explicit integral representations of knot and link invariants of the type of Gauss' integral for the linking numbers of two curves [2]. And second, that perturbative quantization controls gauge invariance for the quantum theory through Becchi-Rouet-Stora (BRS) invariance, which in a sense corresponds to first quantiz-

ing and then constraining, the opposite approach to what is usual in canonical quantization of CS theory [3].

In perturbative quantization, the quantum theory is constructed by demanding it to have certain symmetries. The problem of determining the symmetries that characterize the quantum theory thus becomes a fundamental issue. Classically, the theory has two symmetries: topological invariance or invariance under changes of the spacetime metric, and gauge invariance. Topological invariance is trivially established, for both the classical action [see Eq. (2.2)] and the observables [see Eq. (2.7)] are independent of any metric. However, to later quantize the theory one fixes the gauge and gauge-fixing needs of a choice of metric so that the explicit metric independence of the classical action is lost. This does not spoil classical topological invariance, since the spacetime metric only enters in a BRS exact term and BRS exact terms have no observable meaning. Though, one is left with BRS as the only manifest symmetry of the classical gauge-fixed theory. Not quite. It happens that the gauge-fixed classical action in the Landau gauge has a new symmetry, the so-called Landau gauge supersymmetry [4,5]. This new symmetry has been used in Ref. [6] to prove perturbative finiteness to all orders (see Ref. [7] for an alternative proof), but on the other hand is a symmetry in only the Landau gauge. The purpose of this paper is to study the relevance of this symmetry.

It will turn out that the Landau gauge supersymmetry has no relevance and that, furthermore, it does not play any role in the construction of the quantum theory. We will show this in Sec. II. To actually compute the shift of the bare CS parameter  $k$  within the perturbative framework one has to use a regularization prescription. It hap-

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pens that all BRS-invariant regulators used so far [1,8–11] produce at one loop the same shift as in Eq. (1.1). However, Landau supersymmetric regulators [12] do not. Unfortunately, there is no known regulator preserving both BRS invariance and the Landau gauge supersymmetry simultaneously. In Sec. III we analyze the Landau gauge supersymmetry breaking for a particular BRS invariant regulator [8,9], the only one which has produced as yet a check of the shift in Eq. (1.1) at two loops. Finally, Sec. IV contains our conclusions as well as a discussion of the existence of a unique parametrization for quantum CS theory.

## II. BRS INVARIANCE, THE LANDAU GAUGE SUPERSYMMETRY, AND FINITE RENORMALIZATIONS

The CS action in the Landau gauge for a  $SU(N)$  gauge connection  $A_\mu^a$  on  $\mathbb{R}^3$  reads, in the fundamental representation,

$$S = S_{\text{CS}} + S_{\text{GF}} , \quad (2.1)$$

where  $S_{\text{CS}}$  is the classical CS action

$$S_{\text{CS}} = -\frac{ik}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \left[ \frac{1}{2} A_\mu^a \partial_\rho A_\nu^a + \frac{1}{3!} f^{abc} A_\mu^a A_\rho^b A_\nu^c \right] \quad (2.2)$$

and  $S_{\text{GF}}$  is the Landau gauge-fixing term

$$S_{\text{GF}} = \int d^3x [ -b^a \partial A^a + \bar{c}^a \partial^\mu (D_\mu c)^a ] . \quad (2.3)$$

The parameter  $k$  in Eq. (2.2) is the classical or bare CS parameter. As usual,  $b^a$  denotes the Lagrange multiplier imposing the gauge condition  $\partial A^a = 0$ ,  $c^a$  and  $\bar{c}^a$  are Faddeev-Popov ghosts, and  $D_\mu^{ac} = \delta^{ac} \partial_\mu + f^{abc} A_\mu^b$  is the covariant derivative. The structure constants  $f^{abc}$  are completely antisymmetric and are normalized so that  $f^{acd} f^{bcd} = c_v \delta^{ab}$ . We will keep  $c_v$  in the notation although for  $SU(N)$  one has the simple expression  $c_v = N$ . The action in Eq. (2.1) is invariant under BRS transformations:

$$\begin{aligned} s A_\mu^a &= (D_\mu c)^a , \quad s b^a = 0 , \\ s c^a &= -\frac{1}{2} f^{abc} c^b c^c , \quad s \bar{c}^a = b^a . \end{aligned} \quad (2.4)$$

Note that the gauge-fixing term introduces a metric thus spoiling the metric independence of the CS classical action  $S_{\text{CS}}$ . Classical topological invariance is nevertheless guaranteed by the BRS exactness of  $S_{\text{GF}}$ ,

$$S_{\text{GF}} = - \int d^3x s(\bar{c}^a \partial A^a) ,$$

and the fact that BRS exact quantities are unobservable, i.e., unphysical.

In addition to BRS invariance, the action  $S$  has the two symmetries [4,5]

$$\begin{aligned} v_\mu A_\nu^a &= \frac{4\pi i}{k} \epsilon_{\mu\nu\rho} \partial^\rho c^a , \quad v_\mu b^a = -(D_\mu c)^a , \\ v_\mu c^a &= 0 , \quad v_\mu \bar{c}^a = A_\mu^a , \end{aligned} \quad (2.5)$$

and

$$\begin{aligned} \bar{v}_\mu A_\nu^a &= -\frac{4\pi i}{k} \epsilon_{\mu\nu\rho} \partial^\rho \bar{c}^a , \quad \bar{v}_\mu b^a = \partial_\mu \bar{c}^a , \\ \bar{v}_\mu c^a &= A_\mu^a , \quad \bar{v}_\mu \bar{c}^a = 0 . \end{aligned} \quad (2.6)$$

These two sets of symmetries are indistinctively called Landau gauge supersymmetry. It is important to notice that  $S_{\text{CS}}$  and  $S_{\text{GF}}$  are not separately invariant under  $v_\mu$  nor under  $\bar{v}_\mu$ , but that it is the whole gauge-fixed action  $S$  that is invariant. Furthermore, the Landau gauge supersymmetry is only an invariance of the gauge-fixed classical action in the Landau gauge, never of the Wilson loops (the observables of the theory). To see the latter, we recall the definition of the Wilson loop for a closed curve  $C$ :

$$W(C) = \text{tr} P \exp \left\{ \oint_C A_\mu^a T^a dx^\mu \right\} , \quad (2.7)$$

$T^a$  being the generators of the Lie algebra of the gauge group. It is obvious that  $W(C)$  is not invariant under  $v_\mu$  nor under  $\bar{v}_\mu$ .

Here we want to study the significance of these symmetries for the quantum theory. It is obvious that a quantum CS theory without BRS invariance would not make any sense. On the contrary, one expects the Landau gauge supersymmetry not to have much relevance, despite the fact it was useful in proving perturbative finiteness [6]. We expect the latter on the basis that something that only holds in a particular gauge cannot have much significance. In the sequel we show that one can introduce at will a breaking of the Landau gauge supersymmetry at the quantum level by simply performing finite wave-function renormalizations.

To discuss BRS invariance at the quantum level, we introduce the standard external fields  $J^{a\mu}$  and  $H^a$  coupled respectively to the nonlinear BRS transforms  $s A_\mu^a$  and  $s c^a$  so that the gauge-fixed classical action becomes

$$\Gamma_0 = S_{\text{CS}} + S_{\text{GF}} + S_{\text{EF}} , \quad (2.8)$$

where

$$S_{\text{EF}} = \int d^3x \left[ J^{a\mu} (D_\mu c)^a - \frac{1}{2} f^{abc} H^a c^b c^c \right] .$$

It is well known that symmetries at the quantum level are governed by their corresponding Ward identities so what we need are the Ward identities for the BRS symmetry and the Landau gauge supersymmetries. The Ward identity for the BRS symmetry or BRS identity takes in our notation the form

$$\int d^3x \left[ \frac{\delta\Gamma}{\delta A_\mu^a} \frac{\delta\Gamma}{\delta J^{a\mu}} + \frac{\delta\Gamma}{\delta H^a} \frac{\delta\Gamma}{\delta c^a} + b^a \frac{\delta\Gamma}{\delta \bar{c}^a} \right] = 0, \quad (2.9)$$

where  $\Gamma$  is the effective action. In turn, the Ward identities for the Landau gauge supersymmetries in Eqs. (2.5) and (2.6) read

$$\int d^3x \left[ \frac{4\pi i}{k} \epsilon_{\mu\nu\rho} (\partial^\nu c^a) \frac{\delta\Gamma}{\delta A_\rho^a} + \frac{4\pi i}{k} \epsilon_{\mu\nu\rho} (\partial^\nu J^{a\rho}) \frac{\delta\Gamma}{\delta H^a} - A_\mu^a \frac{\delta\Gamma}{\delta \bar{c}^a} + \frac{\delta\Gamma}{\delta b^a} \frac{\delta\Gamma}{\delta J^{a\mu}} \right] = 0 \quad (2.10)$$

and

$$\int d^3x \left[ \frac{4\pi i}{k} \epsilon_{\mu\nu\rho} (J^{a\nu} - \partial^\nu \bar{c}^a) \frac{\delta\Gamma}{\delta A_\rho^a} - A_\mu^a \frac{\delta\Gamma}{\delta c^a} - (\partial_\mu \bar{c}^a) \frac{\delta\Gamma}{\delta b^a} - H^a \frac{\delta\Gamma}{\delta J^{a\mu}} \right] = \int d^3x \left[ \frac{4\pi i}{k} \epsilon_{\mu\nu\rho} J^{a\nu} \partial^\rho b^a + J^{a\nu} \partial_\mu A_\nu^a - H^a \partial_\mu c^a \right], \quad (2.11)$$

respectively. One also wants the choice of gauge to be preserved by quantization so that one supplements these equations above with the Ward identity

$$\frac{\delta\Gamma}{\delta b^a} + \partial A^a = 0. \quad (2.12)$$

This equation, together with Eq. (2.9), implies that

$$\partial^\mu \frac{\delta\Gamma}{\delta J^{a\mu}} - \frac{\delta\Gamma}{\delta \bar{c}^a} = 0. \quad (2.13)$$

The effective action  $\Gamma$  is an integrated functional of mass dimension three and ghost number zero that depends on the fields  $A_\mu^a$ ,  $b^a$ ,  $c^a$ ,  $\bar{c}^a$ ,  $J^{a\mu}$ , and  $H^a$  and that has local and nonlocal contributions. In perturbation theory,  $\Gamma$  is given by a loop expansion

$$\Gamma = \sum_{n=0}^{\infty} \Gamma_n,$$

where the zero-order contribution  $\Gamma_0$  is the tree-level action in Eq. (2.8) and  $\Gamma_n$  stands for the order  $\hbar^n$  correction. We want to find the most general structure of its local part compatible with Eqs. (2.9)–(2.13). So let us analyze each one of these equations. Equation (2.13) implies that  $\Gamma$  depends on the fields  $J^{a\mu}$  and  $\bar{c}^a$  through the combination  $J^{a\mu} - \partial^\mu \bar{c}^a$ . Equation (2.12) on its own implies that  $\Gamma$  will be of the form  $\Gamma = \bar{\Gamma} - \int d^3x b \partial A$ , with  $\bar{\Gamma}$  an integrated functional with the same mass dimension and ghost number as  $\Gamma$  but independent of  $b^a$ .

The analysis of the BRS identity Eq. (2.9) is more in-

volved. As a first step, it requires showing that it has a solution, or in a more familiar language, that there is no BRS anomaly. That this is the case was proved in Ref. [7]. The local part of  $\Gamma_1$  can be actually constructed using the method of induction and solving the corresponding linearized equation

$$\Delta \Gamma_1 = 0, \quad (2.14)$$

where  $\Delta$  is the Slavnov-Taylor operator:

$$\Delta = \int d^3x \left[ \frac{\delta\Gamma_0}{\delta A_\mu^a} \frac{\delta}{\delta J^{a\mu}} + \frac{\delta\Gamma_0}{\delta J^{a\mu}} \frac{\delta}{\delta A_\mu^a} + \frac{\delta\Gamma_0}{\delta c^a} \frac{\delta}{\delta H^a} + \frac{\delta\Gamma_0}{\delta H^a} \frac{\delta}{\delta c^a} + b^a \frac{\delta\Gamma_0}{\delta \bar{c}^a} \right].$$

This operator is the quantum generalization of the BRS classical operator  $s$  and, as the latter, is nilpotent:  $\Delta^2 = 0$ . We have already said that  $\Gamma_1$  contains local and nonlocal contributions. A thorough study of Eq. (2.14) shows, however, that local contributions in  $\Gamma_1$  decouple from nonlocal ones [9] and gives for the local part of  $\Gamma_1$  the expression [7,9]

$$\Gamma_1^{\text{local}} = \alpha S_{\text{CS}} + \Delta \int d^3x [\beta (J^{a\mu} - \partial^\mu \bar{c}^a) A_\mu^a - \gamma H^a c^a], \quad (2.15)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are arbitrary coefficients of order  $\hbar$ . In what follows we will omit the superscript “local” from the notation. Putting together  $\Gamma_0$  and  $\Gamma_1$  we obtain the effective action up to order  $\hbar$ :

$$\Gamma = -\frac{ik}{4\pi} \int d^3x \epsilon^{\mu\nu\rho} \left[ \frac{1}{2} (1 + \alpha + 2\beta) A_\mu^a \partial_\rho A_\nu^a + \frac{1}{3!} (1 + \alpha + 3\beta) f^{abc} A_\mu^a A_\rho^b A_\nu^c \right] + \int d^3x \left\{ -b^a \partial A^a + (J^{a\mu} - \partial^\mu \bar{c}^a) [(1 - \beta + \gamma) \partial_\mu c^a + (1 + \gamma) f^{abc} A_\mu^b c^c] - \frac{1}{2} (1 + \gamma) f^{abc} H^a c^b c^c \right\}. \quad (2.16)$$

Note that the theory is not finite by power counting so to make explicit computations one has to use a regularization method. As is well known, any regularization method will introduce ambiguities in Green functions which are divergent by power counting, whereas Green functions already convergent by power counting will remain unambiguous. It happens that the only Green functions which diverge by power counting and, hence, the only sources of ambiguities, involve fewer than four fields. We have seen that their generating functional at one loop is given by Eq. (2.16). To find explicit values for the coefficients  $\alpha$ ,  $\beta$ , and  $\gamma$  one may use a BRS invariant regularization method, with different methods yielding in general different values. Recall that the theory being finite, though not by power counting, implies that the  $\alpha$ ,  $\beta$ , and  $\gamma$  are finite after whatever regulator one decides to use is removed.

The structure of  $\Gamma_1$  in Eq. (2.15) shows that there are two types of radiative corrections. We have on the one hand radiative corrections labeled by  $\beta$  and  $\gamma$ ; they correspond to the cohomologically trivial term

$$\Delta X \equiv \Delta \int d^3x [\beta(J^{a\mu} - \partial^\mu \bar{c}^a) A_\mu^a - \gamma H^a c^a] \quad (2.17)$$

and, hence, do not contribute to the vacuum expectation values of the observables. On the other hand, we have the radiative corrections labeled by  $\alpha$ ; they correspond to the gauge invariant quantity  $\alpha S_{CS}$  and contribute to the vacuum expectation values of the observables. The fact that radiative corrections of the first type have the cohomologically trivial form  $\Delta X$  ensures that they can be set to zero by renormalizing only the fields. Indeed, any wave-function renormalization of the form

$$\Phi = Z_\Phi \Phi', \quad (2.18)$$

with

$$Z_A = Z_b^{-1} = 1 - \beta, \quad Z_c Z_{\bar{c}} = 1 + \beta - \gamma,$$

$$Z_H Z_c^2 = 1 - \gamma, \quad Z_J = Z_{\bar{c}},$$

absorbs the contribution  $\Delta X$  to the effective action so  $\Gamma$  in terms of the renormalized fields  $\Phi'$  writes

$$\begin{aligned} \Gamma' = & -\frac{ik(1+\alpha)}{4\pi} \int d^3x \epsilon^{\mu\rho\nu} \left[ \frac{1}{2} A_\mu'^a \partial_\rho A_\nu'^a + \frac{1}{3!} f^{abc} A_\mu'^a A_\rho'^b A_\nu'^c \right] \\ & + \int d^3x [-b'^a \partial A'^a + (J'^{a\mu} - \partial^\mu \bar{c}'^a)(D'_\mu c')^b - \frac{1}{2} f^{abc} H'^a c'^b c'^c], \end{aligned} \quad (2.19)$$

or more simply

$$\Gamma' = \Gamma_0[\Phi', k(1+\alpha)].$$

We denote by  $R'$  the renormalization scheme in Eq. (2.18). Let us stress that in  $R'$  the renormalized CS parameter is equal to the bare one. Equation (2.19) clearly displays that the bare parameter is shifted so that the monodromy parameter becomes  $k(1+\alpha)$ . This is the appealing feature of  $R'$ . Notice that having a renormalized parameter equal to the bare one is not in contradiction with renormalization theory, since CS theory is finite. More generally, in any finite field theory the renormalization scheme  $Z_{\text{fields}} = Z_{\text{parameters}} = 1$  is as good as any other scheme, as opposed to only renormalizable theories, where such a scheme would not give finite renormalized Green functions.

Another important observation [7] concerning the structure of the radiative corrections in Eq. (2.15) is that the metric only enters in the cohomologically trivial term  $\Delta X$ . This means that changes of the metric do not reach the vacuum expectation values of the observables and guarantees topological invariance at the quantum level. In other words, quantum topological invariance follows from quantum BRS invariance.

Local higher-order corrections to  $\Gamma_0$  can be constructed recursively. Local second-order radiative corrections correspond to the solution of the equation  $\Delta'\Gamma_2=0$ , where  $\Delta'$  is the Slavnov-Taylor operator constructed with the action  $\Gamma'$  and the fields  $\Phi'$ . Since

$\Gamma' = \Gamma_0[\Phi', k(1+\alpha)]$ , the operator  $\Delta'$  is obtained from  $\Delta$  by simply replacing the fields  $\Phi$  with their renormalized counterparts  $\Phi'$  and  $k$  with  $k(1+\alpha)$ . This gives for  $\Delta'\Gamma_2=0$  an equation of the form (2.14), whose solution has just been analyzed and which leads to an expression for the local part of the effective action up to second order of the type (2.19). In general, the effective action up to order  $n$  is given by  $\Gamma' = \Gamma_0[\Phi_{(n)}, k(1+\alpha_{(n)})]$ , with the fields  $\Phi_{(n)}$  related to the fields  $\Phi_{(n-1)}$  in the same way as  $\Phi'$  are related to  $\Phi$  in Eq. (2.18) and with  $\alpha_{(n)}$  a power series in  $\hbar$  going up to  $\hbar^n$ . This concludes the analysis of the BRS identity.

We next study the Ward identities for the Landau gauge supersymmetry. The absence of radiative corrections to the ghost two-point Green function in Eq. (2.19) reveals that  $\Gamma'$  is not Landau supersymmetric. The question that arises then is whether there is any field redefinition such that the effective action in terms of the redefined fields satisfies the two Ward identities Eqs. (2.10) and (2.11). In what follows we provide an answer in the affirmative. Any wave-function renormalization

$$\Phi = Z_\Phi \Phi'', \quad (2.20)$$

with

$$Z_A = Z_b^{-1} = 1 - \frac{1}{2}\alpha - \beta, \quad Z_c Z_{\bar{c}} = 1 + \beta - \gamma,$$

$$Z_H Z_c^2 = 1 - \frac{1}{2}\alpha - \gamma, \quad Z_J = Z_{\bar{c}},$$

leads to the renormalized effective action

$$\begin{aligned}
\Gamma'' = \int d^3x \left[ -\frac{ik}{4\pi} \epsilon^{\mu\rho\nu} \left( \frac{1}{2} A''^a_\mu \partial_\rho A''^a_\nu + \frac{1}{3!} f^{abc} A''^a_\mu A''^b_\rho A''^c_\nu \right) - b''^a \partial A''^a \right. \\
\left. + (J''^{a\mu} - \partial^\mu \bar{c}''^a) D''^{ab}_\mu c''^b - \frac{1}{2} f^{abc} H''^{ac}{}_{\mu\nu} c''^b c''^c \right] \\
- \frac{\alpha}{2} \int d^3x \left[ -\frac{ik}{4\pi} \epsilon^{\mu\rho\nu} \frac{1}{3!} f^{abc} A''^a_\mu A''^b_\rho A''^c_\nu + f^{abc} (J''^{a\mu} - \partial^\mu \bar{c}''^a) A''^b_\mu c''^a - \frac{1}{2} f^{abc} H''^{ac}{}_{\mu\nu} c''^b c''^c \right]. \quad (2.21)
\end{aligned}$$

It is straightforward to check that this action satisfies Eqs. (2.9)–(2.12) for the renormalized fields, thus ensuring that  $\Gamma''$  is both BRS invariant and Landau supersymmetric. We will denote the renormalization scheme in Eq. (2.20) by  $R''$ . In this scheme the renormalized parameter is also equal to the bare one,  $k$ . Furthermore, since  $\Gamma'$  in Eq. (2.19) and  $\Gamma''$  in Eq. (2.21) are related by a field redefinition, the vacuum expectation values of the observables computed from both actions (whatever they turn out to be) are the same. Hence, the monodromy parameter in the scheme  $R''$  is also  $k(1+\alpha)$ . In this sense, the shift is still present in the action  $\Gamma''$ , though hidden.

Using different arguments, it has been shown [6] that the most general solution over the space of local integrated functionals of Eqs. (2.9)–(2.12) is precisely the effective action in Eq. (2.21) for arbitrary  $\alpha$ . Our analysis then proves that the Landau gauge supersymmetry is devoid of any meaning, since having a quantum breaking or not having it is a matter of a field redefinition and fields are nothing but nonobservable coordinates in the functional space in which the effective action and the Wilson loops are defined. In a different language, what makes sense are the cohomology classes defined by  $\Delta Y=0$ , with  $Y$  a local integrated functional of mass dimension three and ghost number zero. These cohomology classes are labeled by  $\alpha$  and each one of them contains an infinite number of undistinguishable elements. Imposing the Landau gauge supersymmetry at the quantum level amounts to choosing a particular representative in a class, a choice which is well known to be irrelevant.

### III. BRS-INVARIANT REGULARIZATION AND BROKEN LANDAU SUPERSYMMETRY

In this section we use a BRS invariant regularization method to explicitly illustrate at first order in perturbation theory what we have discussed at all orders in the previous section.

The need for a regularization method comes from the fact that, although CS theory is known to be UV finite, the theory is only renormalizable by power counting. This means that to compute Green functions order by order in perturbation theory, a regularization prescription must be introduced. The regularization method we will use here consists in defining CS theory as the large mass limit of topologically massive Yang-Mills (TMYM) theory, whose action in the Landau gauge has the form [13,14]

$$S_m = S + S_{\text{YM}}, \quad S_{\text{YM}} = \frac{k}{16\pi m} \int d^3x F^a_{\mu\nu} F^{a\mu\nu}, \quad (3.1)$$

with  $S$  the CS action as given in Eq. (2.1),  $F^a_{\mu\nu}$  the field strength of the gauge connection  $A^a_\mu$ , and  $m$  a mass parameter to be sent to infinity at the end of the calculations. We will take  $k > 0$  so that the factor  $e^{-S_m}$  ensures formal convergence of the path integral. The theory defined by  $S_m$  has a finite number of superficially divergent one-particle-irreducible (1PI) Feynman diagrams so the adding of a Yang-Mills term  $S_{\text{YM}}$  to the action  $S$  does not completely regularize CS theory. To take care of the residual divergences we use dimensionally regularization. Our method can then be viewed as a hybrid regularization that combines a higher covariant derivative Yang-Mills term and dimensional regularization. Let us be more precise and spend a few words on the regularized theory.

We would first like to recall that there is a well-known and consistent prescription to deal with the Levi-Civita tensor in dimensional regularization, namely the original prescription of 't Hooft and Veltman [15,16]. Calculations certainly get complicated, since evanescent operators enter in the game, but algebraic consistency (something indispensable in any regularization method [17]) is ensured. The prescription defines the  $D$ -dimensional analogue of  $\epsilon_{\mu\nu\rho}$  as a completely antisymmetric object in its indices which satisfies the properties

$$\begin{aligned}
\epsilon_{\mu_1\mu_2\mu_3}\epsilon_{\nu_1\nu_2\nu_3} &= \sum_{\pi \in S_3} \text{sgn}(\pi) \prod_{i=1}^3 \tilde{g}_{\mu_i\nu_{\pi(i)}}, \\
\epsilon_{\mu_1\mu_2\mu_3}\hat{g}^{\mu_3\mu_4} &= 0. \quad (3.2)
\end{aligned}$$

Here  $g_{\mu\nu} = \tilde{g}_{\mu\nu} \oplus \hat{g}_{\mu\nu}$  is the Euclidean metric in  $D$  dimensions and  $\tilde{g}_{\mu\nu}$  and  $\hat{g}_{\mu\nu}$  its three- and  $(D-3)$ -dimensional projections, respectively, so that  $\tilde{g}_{\mu\nu}\tilde{g}^{\mu\nu}=3$  and  $\hat{g}_{\mu\nu}\hat{g}^{\mu\nu}=D-3$ . Any  $D$ -dimensional vector  $u^\mu$  can be written as  $u^\mu = \tilde{u}^\mu \oplus \hat{u}^\mu$ , where  $\hat{u}^\mu = \hat{g}^{\mu\nu}u_\nu$  and  $\tilde{u}^\mu = \tilde{g}^{\mu\nu}u_\nu$ . Objects with a hat vanish for  $D=3$  and are called evanescent. We stress that this prescription for  $\epsilon_{\mu\nu\rho}$  in  $D$  dimensions is the only known one algebraically consistent; it has proved successful in perturbative computations in a variety of models, including Wess-Zumino-Witten (WZW) models [18] and nonlinear  $\sigma$  models [19].

Armed with this prescription, it is easy to construct a dimensionally regularized TMYM theory that manifestly preserves BRS invariance. One first extends the three-dimensional action  $S_m$  in Eq. (3.1) to  $D$  dimensions, with  $D$  an integer. Next, one obtains the corresponding  $D$ -dimensional Feynman rules. Finally, one promotes  $D$  to a complex variable and defines every  $D$ -dimensional

Feynman integral entering in a Feynman diagram using the dimensional regularization techniques in Ref. [20]. We must emphasize at this point that only the algebraic properties of the objects  $\epsilon_{\mu\rho\nu}$ ,  $\hat{g}_{\mu\nu}$ ,  $\tilde{g}_{\mu\nu}$ , and  $u_\mu$  are retained for complex values of  $D$  [16]. Notice also that invariance of the  $D$ -dimensional action under  $D$ -dimensional BRS transformations, together with the properties of dimensionally regularized integrals, ensures that the formal BRS identities hold for the regularized theory. The latter is the same as saying that TMYM theory dimensionally regularized in this way is manifestly BRS invariant.

Our regularization method thus defines CS theory as the limit  $m \rightarrow \infty$  of the limit  $D \rightarrow 3$  of dimensionally regularized TMYM theory. It is easy to realize that these two limits do not commute and that they must be taken in this order if one wants to define a sensible regularization. Notice that a necessary condition to be able to take the limit  $m \rightarrow \infty$  is that the limit  $D \rightarrow 3$  be finite. If singularities appear as  $D$  goes to 3, it does not make sense

to take  $m \rightarrow \infty$ . It happens that the limit  $D \rightarrow 3$  is free of singularities to all orders in perturbation theory [9]. This does not only permit to take the limit  $m \rightarrow \infty$  but also proves that TMYM theory is finite.

We have anticipated that the definition in Eqs. (3.2) for the  $D$ -dimensional  $\epsilon_{\mu\nu\rho}$  introduces evanescent operators. Let us be more explicit about this. The problem is that the definition in Eqs. (3.2) makes the formal regularized theory invariant under  $\text{SO}(3) \otimes \text{SO}(D-3)$ , rather than under  $\text{SO}(D)$ . As a result, the gauge field free propagator involves objects with carets and tildes in a nontrivial way. To see this, we write the gauge field free propagator  $D_{\mu\nu}(\tilde{p}, \hat{p})$  in full detail (see Ref. [9] for the Feynman rules):

$$D_{\mu\nu}(\tilde{p}, \hat{p}) = \Delta_{\mu\nu}(p) + R_{\mu\nu}(\tilde{p}, \hat{p}) , \quad (3.3)$$

where for simplicity we have dropped color indices and where  $\Delta_{\mu\nu}(p)$  and  $R_{\mu\nu}(\tilde{p}, \hat{p})$  are given by

$$\begin{aligned} \Delta_{\mu\nu}(p) &= \frac{4\pi}{k} \frac{m}{p^2(p^2 + m^2)} (m \epsilon_{\mu\rho\nu} p^\rho + p^2 g_{\mu\nu} - p_\mu p_\nu) , \\ R_{\mu\nu}(\tilde{p}, \hat{p}) &= \frac{4\pi}{k} \frac{m^3}{p^2[(p^2)^2 + m^2 \tilde{p}^2]} \left[ \frac{\hat{p}^2}{p^2 + m^2} \left[ m \epsilon_{\mu\rho\nu} p^\rho + p^2 g_{\mu\nu} + \frac{m^2}{p^2} p_\mu p_\nu \right] + \tilde{p}^2 \hat{g}_{\mu\nu} + \hat{p}_\mu \hat{p}_\nu - p_\mu \hat{p}_\nu - \hat{p}_\mu p_\nu \right] . \end{aligned} \quad (3.4)$$

It is obvious that quantities with carets do not contribute at the tree level, since they vanish at  $D=3$ . This does not imply, however, that they do not contribute at higher orders in perturbation theory, for integration over the internal momenta of a Feynman diagram is prior to taking the limit  $D \rightarrow 3$  and integration may give rise to poles in  $D-3$ . Here we limit ourselves to showing that the caret-ed or evanescent piece  $R_{\mu\nu}(\tilde{p}, \hat{p})$  does not contribute to the limit  $D \rightarrow 3$  of the one-loop diagrams we will compute (see Fig. 2). To this end, we recall [20] that if the integral of an evanescent quantity is convergent by power counting, then its dimensionally regularized integral vanishes as  $D$  approaches the dimensionality of interest, three in our case. Accordingly, it is enough to check that evanescent integrals arising from the diagrams we are interested in are finite by power counting at  $D=3$ . But the latter follows straightforwardly if one takes into account that the UV degree of  $R_{\mu\nu}(\tilde{p}, \hat{p})$  is  $-4$ . [For a proof to all orders in perturbation theory of the no contribution of  $R_{\mu\nu}(\tilde{p}, \hat{p})$  to the limit  $D \rightarrow 3$  of any Green function, see Ref. [9].] We can then use  $\Delta_{\mu\nu}(p)$  as the gauge field free propagator in our calculations. This “effective” propagator could have been derived from the three-dimensional one by promoting the three-momentum to  $D$  dimensions. Despite how appealing this shortcut might look, one has to follow the long road we have followed here if one wants to make sure that the evanescent objects ensuring BRS invariance at the regularized level do not contribute as  $D$  goes to 3.

The one-loop corrections to the vacuum polarization tensor  $\Pi_{\mu\nu}^{ab}(p)$ , to the ghost self-energy  $\Pi^{ab}(p)$  and to the three-vertex  $\Gamma_{\mu\nu\rho}^{abc}$  computed with this regularization prescription are [9]

$$\begin{aligned} \Pi_{\mu\nu}^{ab}(p) &= \frac{7}{3} \frac{c_V}{4\pi} \delta^{ab} \epsilon_{\mu\rho\nu} p^\rho , \quad \Pi(p)^{ab} = \frac{2}{3} \frac{c_V}{k} \delta^{ab} p^2 , \\ \Gamma_{\mu\nu\rho}^{abc} &= 3 \frac{c_V}{4\pi} f^{abc} \epsilon_{\mu\nu\rho} \end{aligned} \quad (3.5)$$

(plus contributions that vanish as  $D$  approaches 3 and  $m$  goes to infinity). Equations (3.5) give for the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  of the previous section the values

$$\alpha = \frac{c_V}{k} , \quad \beta = \frac{2}{3} \frac{c_V}{k} , \quad \gamma = 0 .$$

We thus see that our regularization prescription gives for the shift of the CS bare parameter  $k$  the one-loop result

$$k \rightarrow k + c_V . \quad (3.6)$$

This value for the one-loop shift of the bare CS parameter  $k$  has also been obtained using other regularization methods [1,10,11] and is in accordance with results from canonical quantization [1,21].

Whereas our regularization method manifestly preserves BRS invariance, it explicitly breaks the Landau gauge supersymmetry of Eqs. (2.5) and (2.6). To see the latter, we first have to extend the transformations (2.5)

and (2.6) to  $D$  dimensions and then check if they leave invariant the regularized action. The extension of the transformations  $v_\mu$  and  $\bar{v}_\mu$  to  $D$  dimensions is trivially achieved by using the  $D$  dimensional  $\epsilon_{\mu\nu\rho}$  defined earlier and by regarding all functions and fields as defined on  $\mathbb{R}^D$ . It is then very easy to see that the gauge-fixed CS action in  $D$  dimensions is invariant under the  $D$  dimensional  $v_\mu$  and  $\bar{v}_\mu$  but that  $S_{YM}$  is not (see below). Hence, the regularized theory is not Landau supersymmetric. The question that then arises is whether the breaking remains after the regulator is removed. We next show that is indeed the case.

Consider a generic function  $F(\Phi)$  of the fields  $\Phi = \{A_\mu^a, b^a, c^a, \bar{c}^a\}$ . Under an infinitesimal transformation  $\Phi \rightarrow \Phi + \delta\Phi$  of Jacobian equal to one, the following identity holds in the Euclidean formalism:

$$\left\langle \left( \frac{\partial F(\Phi)}{\partial \Phi} - \frac{\delta S_m[\Phi]}{\delta \Phi} F(\Phi) \right) \delta \Phi \right\rangle = 0. \quad (3.7)$$

For  $F(\Phi) = A_\mu^a(x) \bar{c}^b(y)$  and the transformations in Eq. (2.5), Eq. (3.7) reads

$$\langle A_\mu^a(x) A_\nu^b(y) \rangle = \frac{4\pi i}{k} \epsilon_{\mu\rho\nu} \langle \partial_x^\rho c^a(x) \bar{c}^b(y) \rangle + \langle A_\mu^a(x) \bar{c}^b(y) v_\nu S_{YM} \rangle, \quad (3.8)$$

where we have used that  $v_\nu S = 0$ . In the following we explicitly check up to first order in perturbation theory that the identity (3.8) holds in the limit  $D \rightarrow 3$ ,  $m \rightarrow \infty$ . It will appear that the second term on the right-hand side (RHS) gives nonvanishing quantum corrections without which the identity is not satisfied, thus showing that the supersymmetry remains broken after the regulating parameters are removed.

We start by computing  $v_\nu S_{YM}$ . After some algebra we obtain that

$$v_\nu S_{YM} = \mathcal{O}_\nu^{(0)} + \mathcal{O}_\nu^{(1)} + \mathcal{O}_\nu^{(2)}, \quad (3.9)$$

with

$$\begin{aligned} \mathcal{O}_\nu^{(0)} &= -\frac{i}{m} \int d^D x \epsilon_{\nu\mu\rho} (\partial^\rho c^a) \square A^{a\mu}, \\ \mathcal{O}_\nu^{(1)} &= -\frac{i}{m} f^{abc} \int d^D x \epsilon_{\nu\mu\rho} (\partial^\rho c^a) \\ &\quad \times [(\partial A^b) A^{c\mu} + 2 A_\sigma^b (\partial^\sigma A^{c\mu}) \\ &\quad - A_\sigma^b (\partial^\mu A^{c\sigma})], \\ \mathcal{O}_\nu^{(2)} &= \frac{i}{m} f^{abc} f^{bde} \int d^D x \epsilon_{\nu\mu\rho} (\partial^\rho c^a) A_\sigma^c A^{d\sigma} A^{e\mu}. \end{aligned}$$

The operators  $\mathcal{O}_\nu^{(0)}$ ,  $\mathcal{O}_\nu^{(1)}$ , and  $\mathcal{O}_\nu^{(2)}$  have in momentum space the Feynman rules listed in Fig. 1. Calling  $G_{\mu\nu}(p)$

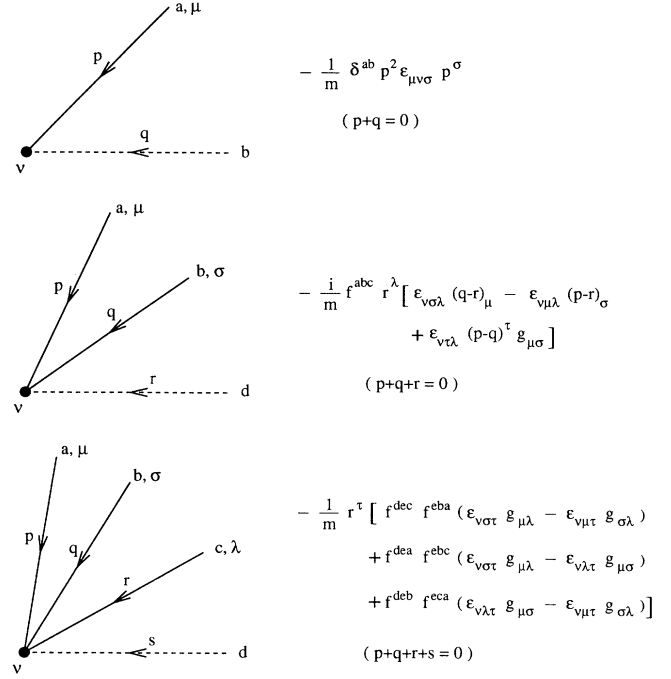


FIG. 1. Feynman rules for the operators  $\mathcal{O}_\nu^{(0)}$ ,  $\mathcal{O}_\nu^{(1)}$ , and  $\mathcal{O}_\nu^{(2)}$ .

and  $G(p)$  to the two-point Green functions of the gauge and ghost fields, the identity in Eq. (3.8) can be recast in momentum space as

$$G_{\mu\nu}(p) = \frac{4\pi}{k} \epsilon_{\mu\rho\nu} p^\rho G(p) + G_{\mu\rho}(p) \Omega_{\nu}^{\rho}(p) G(p), \quad (3.10)$$

where  $\Omega_{\nu}^{\rho}(p)$  is the 1PI Green function associated with the second term on the RHS in Eq. (3.8). From a loop-wise expansion we have

$$\begin{aligned} G_{\mu\nu}(p) &= \Delta_{\mu\nu}(p) + \Delta_{\mu\sigma}(p) \Pi^{\sigma\rho}(p) \Delta_{\rho\nu}(p) + \mathcal{O}(1/k^3), \\ G(p) &= \Delta(p) + \Delta(p) \Pi(p) \Delta(p) + \mathcal{O}(1/k^2), \\ \Omega_{\mu\nu}(p) &= \Omega_{\mu\nu}^{(0)}(p) + \Omega_{\mu\nu}^{(1)}(p) + \mathcal{O}(1/k^2), \end{aligned} \quad (3.11)$$

with  $\Delta_{\mu\nu}(p)$  as in Eq. (3.4),  $\Delta(p) = -1/p^2$  the ghost free propagator, and  $\Pi^{\sigma\rho}(p)$  and  $\Pi(p)$  given in Eq. (3.5). Inserting Eqs. (3.11) in Eq. (3.10) and identifying coefficients in  $1/k$ , we obtain

$$\Delta_{\mu\nu}(p) = \frac{4\pi}{k} \epsilon_{\mu\rho\nu} p^\rho \Delta(p) + \Delta_{\mu\rho}(p) \Omega_{\nu}^{(0)\rho}(p) \Delta(p) \quad (3.12)$$

to order one (tree level), and

$$\begin{aligned} \Delta_{\mu\sigma}(p) \Pi^{\sigma\rho}(p) \Delta_{\rho\nu}(p) &= \frac{4\pi}{k} \epsilon_{\mu\rho\nu} p^\rho \Delta(p) \Pi(p) \Delta(p) + \Delta_{\mu\sigma}(p) \Omega_{\nu}^{(1)\sigma}(p) \Delta(p) \\ &\quad + \Delta_{\mu\sigma}(p) \Pi^{\sigma\rho}(p) \Delta^{\rho\gamma}(p) \Omega_{\gamma\nu}^{(0)}(p) \Delta(p) + \Delta_{\mu\sigma}(p) \Omega_{\nu}^{(0)\sigma}(p) \Delta(p) \Pi(p) \Delta(p) \end{aligned} \quad (3.13)$$

to order two (one loop). The identity in Eq. (3.12) relates the tree-level gauge field and ghost propagators at finite  $m$ . From the Feynman rules in Fig. 1 it follows that in the limit  $m \rightarrow \infty$  the second term on the RHS vanishes, whereas the first one reproduces the CS gauge field free propagator. Showing that Eq. (3.13) is indeed satisfied requires more discussion. The explicit expressions of  $\Pi_{\mu\nu}(p)$  and  $\Pi(p)$  in Eqs. (3.5), together with the Feynman rules in Fig. 1, imply that the third and fourth terms on the RHS are finite and of order  $1/m$  so that they vanish when  $D \rightarrow 3$ ,  $m \rightarrow \infty$ . Equation (3.13) thus reduces in the limit  $D \rightarrow 3$ ,  $m \rightarrow \infty$  to

$$\Delta_{\mu\sigma}(p)\Pi^{\sigma\rho}(p)\Delta_{\rho\nu}(p) = \frac{4\pi}{k}\epsilon_{\mu\rho\nu}p^\rho\Delta(p)\Pi(p)\Delta(p) + \Delta_{\mu\sigma}(p)\Omega^{(1)\sigma}{}_\nu(p)\Delta(p). \quad (3.14)$$

In this equation everything is known except for  $\Omega_{\mu\nu}^{(1)}(p)$ , whose limit  $D \rightarrow 3$ ,  $m \rightarrow \infty$  we compute next.

There are five Feynman diagrams that contribute to  $\Omega_{\mu\nu}^{(1)}(p)$  (see Fig. 2). All Feynman integrals arising from these graphs are of the form

$$I(p, m) = \int d^D q \frac{m^r M(q)}{\prod_i (l_i^2 + m_i^2)^{s_i}}, \quad r, s_i \in \mathbb{N},$$

where  $M(q)$  is a monomial of degree  $n_q$  in the components of the integrated momentum  $q$ , the vectors  $l_i$  are linear combinations of  $q$  and the external momenta  $p_1, \dots, p_E$ , and the masses only take on two values,  $m_i = 0$  and  $m_i = m > 0$ . The external momenta are assumed to lie in a bounded subdomain of  $\mathbb{R}^D$ . As we have already said, we first have to take the limit  $D \rightarrow 3$  of  $I(p, m)$  and then  $m \rightarrow \infty$ . The limit  $D \rightarrow 3$  is always finite, for in dimensional regularization the integral  $I(p, m)$  is finite as  $D$  approaches  $l$  for  $l$  odd, even when  $I(p, m)$  is divergent by power counting [22]. This guarantees that no poles appear when the limit  $D \rightarrow 3$  is taken. To compute the large  $m$  limit of  $I(p, m)$  at  $D = 3$ , hence of the diagrams we are interested in, we use two vanishing theorems. Here we limit ourselves to state them. Their proof and generalization to higher orders in perturbation theory can be found in Ref. [9]. Denoting by  $d$  the mass dimension of  $I(p, m)$  and introducing the notation  $[n] = 0$  for  $n$  even and  $[n] = 1$  for  $n$  odd, the theorems say that

**Theorem 1.** If  $I(p, m)$  is infrared convergent by power counting,  $d < 0$  and  $\alpha m - 2\sum_i \beta_i m_i < 0$ , then  $I(p, m)$  vanishes when  $m$  goes to  $\infty$ .

**Theorem 2.** If  $I(p, m)$  is absolutely convergent by power counting for exceptional configurations of the external momenta and  $[n_q] > d$ , then  $I(p, m) \rightarrow 0$  as  $m \rightarrow \infty$ .

After taking the limits  $D \rightarrow 3$ ,  $m \rightarrow \infty$  and using the theorems, we obtain for the diagrams in Fig. 2 the following results:<sup>1</sup>

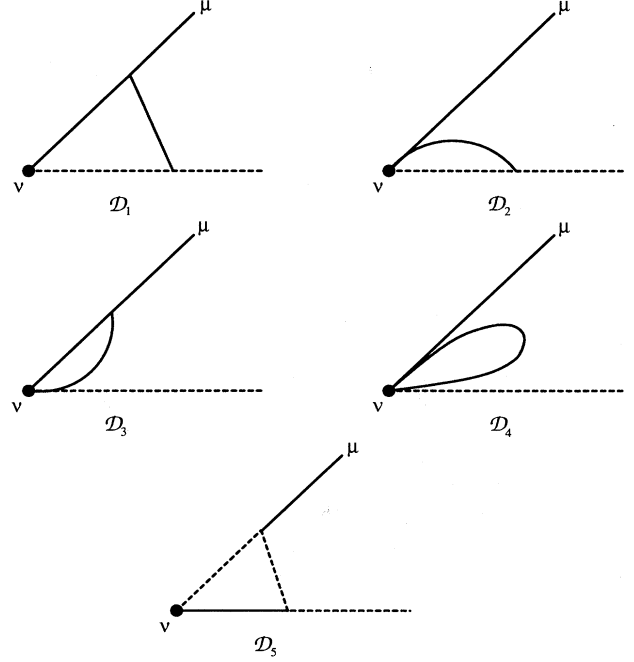


FIG. 2. Feynman diagrams contributing to  $\Omega_{\mu\nu}^{(1)}(p)$ .

$$\mathcal{D}_1 = \frac{c_V}{k} \left( -\frac{13}{15} p^2 g_{\mu\nu} + \frac{19}{15} p_\mu p_\nu + m \epsilon_{\mu\nu\rho} p^\rho \right),$$

$$\mathcal{D}_2 = \frac{c_V}{k} \left( \frac{16}{15} p^2 g_{\mu\nu} - \frac{18}{15} p_\mu p_\nu - \frac{2}{3} m \epsilon_{\mu\nu\rho} p^\rho \right),$$

$$\mathcal{D}_3 = \frac{c_V}{k} \left( \frac{8}{3} p^2 g_{\mu\nu} - \frac{8}{3} p_\mu p_\nu - 2m \epsilon_{\mu\nu\rho} p^\rho \right),$$

$$\mathcal{D}_4 = \frac{c_V}{k} \frac{4}{3} m \epsilon_{\mu\nu\rho} p^\rho,$$

$$\mathcal{D}_5 = \frac{c_V}{k} \left( \frac{2}{15} p^2 g_{\mu\nu} - \frac{6}{15} p_\mu p_\nu + \frac{1}{3} m \epsilon_{\mu\nu\rho} p^\rho \right).$$

Any other contribution vanishes as  $D$  goes to 3 and  $m$  approaches infinity. Summing over diagrams we finally have

$$\Omega_{\sigma\nu}^{(1)}(p) = 3 \frac{c_V}{k} (p^2 g_{\sigma\nu} - p_\sigma p_\nu). \quad (3.15)$$

Note that contributions of order  $m$  from individual diagrams cancel when summing over diagrams, thus making the limit  $m \rightarrow \infty$  well defined. From Eqs. (3.5) and (3.15) it follows that the identity (3.14) is verified. It is very important to realize that were it not for the nonvanishing contribution  $\Omega_{\sigma\nu}^{(1)}(p)$ , the identity (3.14) would not hold. Recalling that  $\Omega_{\sigma\nu}^{(1)}(p)$  had its origin in the supersymmetry-breaking term in the regularized action, we conclude that the supersymmetry remains broken after the regulating parameters are removed and that it is precisely the breaking what is required to have the identity (3.14) satisfied. This is not peculiar of the regularization method used here but has also been observed [24] for a hybrid regulator consisting of a higher covariant

<sup>1</sup>The algebra was performed with the help of the symbolic language REDUCE [23].



derivative term of the form  $(DF)^2$  plus a Pauli-Villars [10] regulator.

The same pattern occurs for the Landau gauge supersymmetry in Eq. (2.6). If in Eq. (3.7) we take  $F(\Phi) = A_\mu^a(x)c^b(y)$  and the transformation  $\bar{v}_\mu$  in Eqs. (2.6), we get the identity

$$\langle A_\mu^a(x) A_\nu^b(y) \rangle = \frac{4\pi i}{k} \epsilon_{\mu\rho\nu} \langle \partial_x^\rho c^a(x) \bar{c}^b(y) \rangle + \langle c^a(x) A_\nu^b(y) \bar{v}_\mu S_{YM} \rangle. \quad (3.16)$$

This identity can be analyzed in exactly the same way as the one in Eq. (3.8). As a matter of fact, both identities have the same form in momentum space, namely Eq. (3.10). One can think of the identity Eq. (3.14) as a consistency check for the one-loop corrections to the vacuum polarization tensor and to the ghost self-energy in Eq. (3.5). In a similar way one can check the value for the one-loop correction  $\Gamma_{\mu\nu\rho}^{abc}$  to the three vertex. In this case, it is enough to take  $F(\Phi) = A_\mu^a(x) A_\nu^b(y) \bar{c}^c(z)$  and the transformation  $v_\rho$  in Eq. (2.5).

#### IV. CONCLUSIONS

We have explicitly shown in Sec. II that the Landau gauge supersymmetry of CS theory [4–6] does not have any significance. We have done this by proving that having a quantum breaking of the supersymmetry or not having it is only a question of a wave-function renormalization which does not affect the vacuum expectation values of the observables. Moreover, we have given two expressions  $\Gamma'$  and  $\Gamma''$  for the local part of the renormalized effective action, both yielding the same vacuum expectation values of the Wilson loops (whatever those turn out to be), but one of them ( $\Gamma''$ ) being Landau supersymmetric and the other one ( $\Gamma'$ ) not.

This observation, combined with the fact that topological invariance is recovered from BRS invariance and the prediction of a shift on the grounds of only BRS invariance [see, e.g., Eq. (2.16)], leaves us with BRS as the only fundamental symmetry of the theory.

To compute the actual value of the shift of the bare CS parameter, a regularization prescription is needed if one insists in employing Feynman diagrams. Using a regularization prescription manifestly preserving the Landau supersymmetry is of no importance, as far as it preserves BRS invariance. The reason is that any BRS invariant regularization prescription will yield an action of the form in Eq. (2.16), which can always be recast in the Landau supersymmetric fashion (2.21), both actions being physically undistinguishable.

In Sec. III we have provided an example of a BRS invariant regularization method that breaks the Landau gauge supersymmetry and we have checked that the latter supersymmetry remains broken after the regulating parameters are removed. The shift of the bare CS parameter as computed with this method is  $k \rightarrow k + c_V$ , in agreement with results from canonical quantization. In Table I we collect in units of  $c_V/k$  the one-loop results for  $\alpha$ ,  $\beta$ , and  $\gamma$  in Eq. (2.16) as computed with all BRS in-

TABLE I. Values in units of  $c_V/k$  of  $\alpha$ ,  $\beta$ , and  $\gamma$  in Eq. (2.15) provided by all BRS invariant regularization methods used as yet.

Regularization method	$\alpha$	$\beta$	$\gamma$
Method in Sec. III	1	2/3	0
$\eta$ -function regularization [1]	1	0	0
Higher covariant derivatives + Pauli-Villars [25]	1	2/9	0
Geometric regularization [11]	1	$4I_n/3\pi$	$\dots$

variant regulators tried so far in CS theory.<sup>2</sup>

As can be seen, different BRS invariant regularization methods give different values for  $\beta$  and  $\gamma$  but the same value for the shift  $\alpha$ . This uniqueness for the value of  $\alpha$  for all BRS invariant regulators tried as yet suggests parametrizing the quantum theory in terms of the bare parameter:

$$k_{\text{renormalized}} = k_{\text{bare}} = k.$$

The idea behind this parametrization is that the quantum theory is unambiguously constructed by BRS invariance, if preserved at the regularized level. Notice that such a parametrization would be nonsensical if two different BRS-preserving regulators yielded different values for  $\alpha$ , but the results in the table show that for all BRS invariant regulators tried to date this is not the case. CS theory thus gives a concrete realization of the idea that, in a finite theory, the bare parameters constitute the right parametrization of the quantum theory, provided one uses regulators preserving the fundamental symmetries of the theory [26].

The agreement on the value of  $\alpha$  for different BRS invariant regulators cannot be explained within the framework of local perturbative renormalization theory [27], for, according to its principles, the ambiguities introduced by any regularization method should reach the value of  $\alpha$ . Note also that local perturbative renormalization theory does not contemplate the idea of a preferred parametrization. Any argument aiming to choosing a particular parametrization has to be found outside this framework. Here we have used the argument of the symmetries characterizing the theory.

It would be desirable to learn whether the one-loop agreement of the table holds at higher orders. We con-

<sup>2</sup>The values given here for higher covariant derivatives plus Pauli-Villars fields are those computed in Ref. [25] rather than those in Ref. [10], where strictly speaking only Pauli-Villars fields and no higher covariant derivative terms are used. Geometric regularization makes use of ghost generations different from the standard Faddeev-Popov ones, so only the pure gauge sector of the renormalized effective action can be compared. The quantity  $I_n$  is defined as

$$I_n = \int_0^\infty dp \frac{(1+p^2)^n}{1+p^2(1+p^2)^{2n}},$$

with  $n > 1$  an integer.

ture that this is the case. Unfortunately, no comparison is possible, since so far only the regularization method proposed here has produced a two-loop computation of the shift [9], with the result that there is no second-order correction to the one-loop result, in agreement with canonical quantization.

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